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MOTION OF A GRANULAR MATERIAL DISCHARGING THROUGH AN ORIFICE

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An experimental investigation is presented concerning particle motion in a column of granular material. The effect of gas flow on the motion of material discharging through an orifice is studied.

When a granular material discharges through an orifice, not all of it starts to move at the same time. Instead, motion occurs only in a definite region above the outlet orifice. This region has the shape of an ellipsoid of revolution [1], whose volume is

$$V_p = 1.57\, H_p \left[\frac{H_p^2}{3} (1 - \varepsilon^2) + \frac{d_0^2}{4} \right]. \tag{1}$$

Within this region (Fig. 1a) the voidage increases, creating dynamic porosity. When particles discharge from an infinite medium, the total additional voidage (compared with the normal porosity of the compact layer) in the dynamic ellipsoid is quantitatively equal to the volume of granular material discharged from the container, and this characterizes the volume of the discharge ellipsoid (Fig. 1a).

The dynamic and discharge ellipsoids develop simultaneously, the volume of the first being considerably greater than that of the second [1, 2].

In an infinite medium these ellipsoids grow without limit [1]. If the column is of finite height and bounded by side walls (bins, silos, etc.), the dynamic ellipsoid may reach the surface, forming a funnel [3]. In this case the height of the discharge ellipsoid increases until it reaches the bottom of the funnel; then it begins to decrease as the material discharges, vanishing completely when discharge stops.

We have investigated the discharge of granular materials (see table) from a continuously replenished column of finite height and width (Fig. 1b). The cases examined are free discharge through an orifice (continuously operating hopper), and discharge in a gas counterflow (shaft furnaces, driers, and so on).

The tests were made using hemicyclindrical vessels, diameter 0.14, 0.21, and 0.29 m, with flat bottoms and transparent front walls. The material was discharged through orifices of semicircular cross section, from 0.02 to 0.07 m in diameter, the height of the column was kept constant at 0.7 m. The nature of particle motion was followed visually through the transparent wall of the model from the vertical displacement of bands of homogeneous colored material which was fed periodically into the vessel in thin layers.

Properties of the Granular Materials Used in the Tests

Material	Particle size, m	Mean diameter d_k, m	Bulk weight $\gamma \cdot (9.8)^{-1}$, N/m²	Natural angle of repose β, deg-min	Mobility η [14]
Friable materials					
fine coke	0.002—0.003	0.0025	480	37—50	0.237
chamotte	0.001—0.002	0.0015	990	35—00	0.270
chamotte	0.002—0.003	0.0025	970	35—45	0.264
chamotte	0.003—0.005	0.0040	920	37—05	0.250
chamotte	0.005—0.008	0.0065	900	38—00	0.235
agglomerate	0.002—0.003	0.0025	1680	34—30	0.277
Chamotte granules	0.003—0.005	0.0040	1080	32—35	0.293

The motion of granular material in a cylindrical container resulting from the discharge of particles through an outlet orifice is called "discharge of the second type" [4]. As material discharges through the orifice a region of loosened particles forms above it. The internal friction coefficient in this region is much reduced, which leads to disturbance of the state of equilibrium of the particles. Under the action of the weight of the column, displacement of the particles occurs along slip planes inclined at an angle α to the plane of the orifice, and the whole column begins to subside. This is due to the relatively low value of the external friction compared to internal friction.

The discharge of granular material through an orifice is discrete in nature. Variations of the thrust forces between particles in the dynamic region lead to the formation and disintegration of dynamically unstable arches of particles above the discharge orifice [5-7]. The geometric shape and strength of the arches are determined by the physical properties of the material together with the dimensions of both the vessel and the discharge orifice. The presence of unstable arches above the orifice during discharge reduces the vertical pressure of the column at the base of the vessel and correspondingly increases the horizontal pressure on the side walls [8-10].

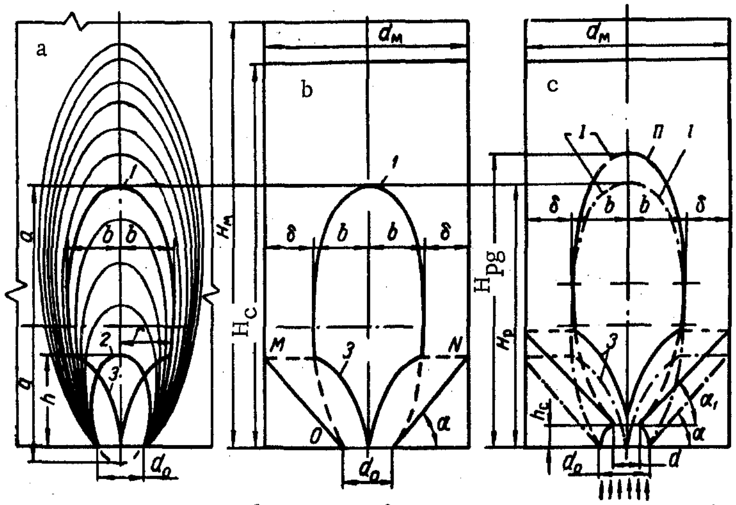

Fig. 1. Motion of particles of granular material above discharge orifice. a) Discharge from an infinite layer; b) from a finite continuously replenished column; and c) from a finite replenished column under conditions of gas counterflow (I—is free discharge, II—discharge in gas counterflow): 1) dynamic ellipsoid; 2) discharge ellipsoid; 3) funnel.

In the discharge of granular materials from finite containers, as in discharge from an infinite medium [1, 2], a dynamic ellipsoid is formed. Its development ends before the apex of the ellipsoid reaches the surface of the finite replenished column. The dimensions of the ellipsoid will remain constant if the rates of descent of the column and upward growth of the ellipsoidal region along the vertical axis are equal (Fig. 1b).

Material is discharged from a moving finite column not only from the dynamic ellipsoid itself [1, 2] but also from the periphery of the column. Particles descending along the walls of the vessel reach the horizontal level MN (Fig. 1b) and are then squeezed into the dynamic ellipsoid, moving toward the discharge orifice. According to test data, the amount of material discharging from the periphery is approximately three times greater than that leaving the ellipsoid

In the case of prolonged discharge the motion of particles from the walls of the vessel to the orifice is accomplished by slippage along the generatrix MO (Fig. 1b) of the truncated cone formed by the dense layer of material around the orifice.

The absolute values of the principal parameters of the dynamic ellipsoid depend upon the relationship between column height and the diameters of vessel, particles, and orifice (Fig. 2). In a moving replenished column, as d_K increases the height H_e (and therefore V_e) diminishes, as do W_0, W_c, and α; the quantity ξ, on the other hand, decreases. The velocities W_0 and W_c depend directly on d_0, while H_p, V_p, α, and ξ are inversely proportional to the same parameter. For constant d_K and d_0, an increase in d_M results in a proportional decrease in both W_c and ξ and an increase in H_p, and V_p, and α. The exception is W_0, the value of which remains constant for a given orifice irrespective of the column height, a fact which has also been noted in [11-13] and elsewhere. The constancy of mass flow rate and the magnitude of the pressure are connected with the creation of dynamically unstable arches above the orifice. The velocity acquired by the particles in free fall from the surface of a crumbling arch to the plane of the orifice [11] determines the value and the constancy of the particle discharge rate from the orifice.

Analysis of the experimental data gives the following expression for the free flow rate of granular material through an orifice:

$$W_0 = 6.67\eta\gamma\sqrt{gd_0^3}(0.403\,d_0^{1.1} - d_\kappa). \qquad (2)$$

The mobility $\eta = 1 + 2f^2 - 2f\sqrt{1+f^2}$ [14], bulk weight x, and particle size in Eq. (2) characterise the dependence of the flow rate on the physical properties of the granular material.

The results of the tests show that the height of the ellipsoid increases with increase in vessel diameter, while with increase in orifice diameter the ellipsoid appears to sink down into the orifice. Reduction in the value of $(d_m - d_0)/2$ cause a corresponding reduction in the height and volume of the discharge funnel, and this, in turn, leads to a lowering of the horizon MN of contact between the descending column and the motionless dense layer around the orifice (Fig. 1b). The lowering of MN correspondingly increases the volume and the pressure of the moving material on the layer of motionless particles. As a result the height of the ellipsoid and its eccentricity decrease proportionately.

Statistical analysis of the test data for the particles investigated gives an equation for the height of the dynamic ellipsoid in a moving column of granular material, under conditions of long-time free flow of particles of irregular shape through a circular orifice:

$$H_p = K d_M^{2n} / d_0^{2n-1},$$

where for

$$\frac{d_M^2}{d_0^2} < 20 \begin{cases} K_1 = 0.1646/d_K^{0.309} \\ n_1 = 1.045 + 0.136 \lg d_K, \end{cases} \tag{3}$$

and for

$$\frac{d_M^2}{d_0^2} > 20 \begin{cases} K_2 = 0.571/d_K^{0.163} \\ n_2 = 0.638 + 0.027 \lg d_K. \end{cases}$$

Under the conditions of the experiments the minor axis 2c of the ellipsoid remained practically unchanged, and the reduction in volume of the dynamic region as d_0 increased was mainly due to a decrease in the major axis 2a. The equation for the volume of the dynamic ellipsoid in dimensionless form is:

$$\frac{V_p}{d_0^3} = K' \left(\frac{d_M}{d_0}\right)^{2m},$$

where for

$$\frac{d_M^2}{d_0^2} < 20 \begin{cases} K_1' = 0.0093/d_K^{0.471} \\ m_1 = 2.342 + 0.283 \lg d_K, \end{cases} \tag{4}$$

and for

$$\frac{d_M^2}{d_0^2} > 20 \begin{cases} K_2^1 = 0.0839/d_K^{0.126} \\ m_2 = 1.6067 + 0.0166 \lg d_K. \end{cases}$$

Values calculated from (3) and (4) are correct to ± 3%. No tests were made for d_M^2/d_0^2 less than 5. According to [12], however, it is to be expected that, in the limiting case of $d_M = d_0$, the separation of particles from a dense column of granula material will also occur along a surface in the shape of a spherical arch whose curvature decreases as the particle diameter increases.

Using Eqs. (3), (4) and the equation for the eccentricity of the dynamic ellipsoid [1]

$$\varepsilon = \left\{1 - \left[0.75\left(2.547\frac{V_p}{d_0^3} - \frac{H_p}{d_0}\right)\right] \Big/ \left(\frac{H_p}{d_0}\right)^3\right\}^{1/2},$$

we obtain the relations shown in dimensionless coordinates in Fig. 2. From these relations the parameters of the dynamic ellipsoids may be determined, without recourse to experiment, for granular materials of given particle size discharging from cylindrical vessels.

It has been established from tests on granular materials of varying density (fine coke, chamotte, agglomerate), for fixed particle size and other basic parameters, that a change in the bulk weight of the material does not produce a marked change in the dimensions of the dynamic ellipsoids. Change of particle shape does have an appreciable effect. In tests on granules specially prepared from chamotte with particle size and bulk weight approximately the same as chamotte particles of irregular shape (see table), the value of H_p/d_0 proved to be roughly 1.5 times less than when chamotte crumbs were used. The fact that the particles were of regular spherical shape reduced internal friction, increased the packing density of the granules in the moving column, and improved the mobility of the material.

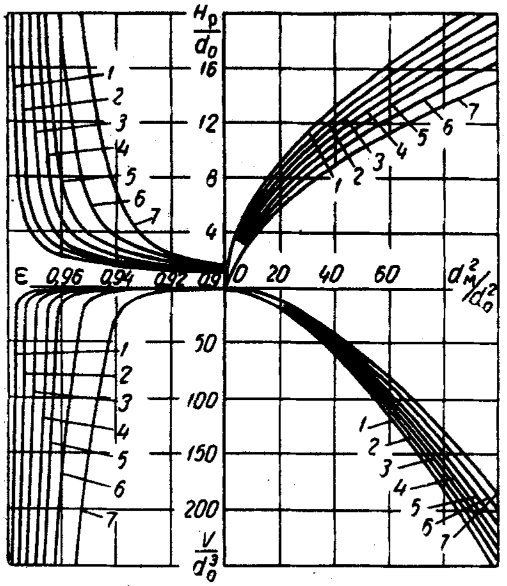

Fig. 2. Relations between principal parameters of dynamic ellipsoid, vessel, and particles. Particle sizes: 1) 0.0015 m; 2) 0.0025; 3) 0.0040; 4) 0.0065; 5) 0.010; 6) 0.020; 7) 0.040.

There are many forms of industrial heat-exchanger apparatus in which particles discharge through an orifice in a gas or air counterflow. Observations of the motion of loose materials in a cylindrical vessel due to discharge of particles through an orifice in an air counterflow have shown that dynamic ellipsoids (Fig. 1c) are also formed in the mass of material above the orifice. However, the discharge of particles retarded by a gas flow differs from free discharge (Fig. 1b). The upward gas flow prevents the discharge of low-velocity particles, causing the formation of an open spherical arch above the orifice (Fig. 1c). An increase of both flow rate and air pressure reduces the effective diameter of the orifice and raises it, together with the walls of the spherical annulus, to a height h_c above its initial level (Fig. 4a). The discharge angle α is then reduced. The relation between the contraction of the effective orifice and the increase of gas flow velocity is given by $d = d_0/0.042W_g^2 + 1$.

A decrease in the diameter of the effective orifice entails a reduced discharge rate (Fig. 3) and a reduced rate of descent of the column. An increased practicle diameter for the same gas flow rate leads to a reduced rate of discharge. Substitution of d in the free flow equation (2) taking into account a coefficient ψ characterizing the cleanness of the edge of the effective orifice ($\psi = 1 - 0,275 W_g$) allows an approximate determination of the flow rate of granular material through an orifice in a gas counterflow.

The gas flow leads to a redistribution within the vessel of the regions of preferred material flow. A slight air flow (up to formation of the walls of an open spherical arch) increases the mobility and the relative rate of discharge of particles moving to the orifice from the periphery of the vessel. With an increase in gas flow, the resistance to discharge of particles from the orifice increases. Formation of spherical walls reduces the diameter d of the orifice and sharply reduces the mobility and the flow of material out of the layer surrounding the ellipsoid. In this case, however, in spite of the over-all reduction in flow rate, the relative rate of discharge of particles directly from the dynamic ellipsoid increases. The formation of an open arch above the orifice raises the dynamic ellipsoid vertically along the orifice axis. The minor axis 2b of the ellipsoid remains practically unchanged. The height of the dynamic ellipsoid increases with increasing gas flow rate roughtly by the height h_c of the effective orifice d formed by spherical zone of stationary particles around the orifice d_0. The volume of the ellipsoid is reduced somewhat due to the reduction in the diameter of the effective orifice.

Further increases of counter flow rate or air pressure lead to closing of the spherical walls of the dome above the orifice (Fig. 1c). The material located above the surface of this dome remains in a loosened condition. The gas pressure ΔP_1 (Fig. 4b) is spent in overcoming the resistance of the column of loose material and the pressure of the flow particles discharging through the orifice. Under the test conditions $d_0 = 0.005$ M; $d_M = 0.21$ M), the air pressure counteracting the discharge of particles in the plane of the orifice at hang-up is $\Delta P_3 = \Delta P_1 - \Delta P_2 = H/M^2$. Calculations show that the measure counterpressure is capable of balancing at most only the weight of particles filling the space below the spherical arch formed above the orifice. This conclusion agrees with the measurements obtained by other methods in [15].

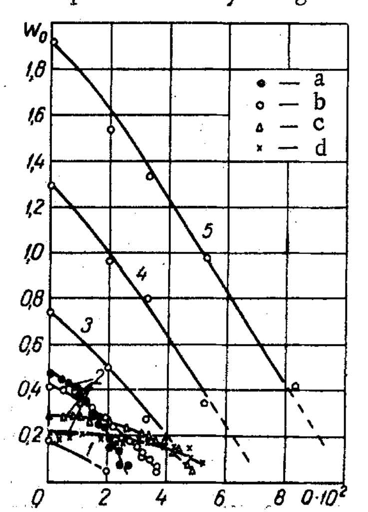

Fig. 3. Variation of flow rate of granular material falling through an orifice as a function of the air flow rate: 1) $d_0 = 0.03$ m; 2) 0.04; 3) 0.05; 4) 0.06; 5) 0.07 for $d_M = 0.21$ m and $d_k = 0.0025$ m; a) $d_k = 0.0015$ m; b) 0.0025; c) 0.0040; d) 0.0065 for $d_0 = 0.040$ m; the broken lines denote the region of unstable discharge and hand-up due to closing of arch above orifice.

In addition to the previously noted discrepancies between the experimental and calculated values of the horizontal and vertical pressure on the walls and base of the vessel [8-10, 15], it has been established that the calculated pressure of the loose material on the area of the orifice, as given by Jansen's formula, is also several times greater than the value found experimentally. This is related to the formation of dynamically unstable arches over the discharge orifice.

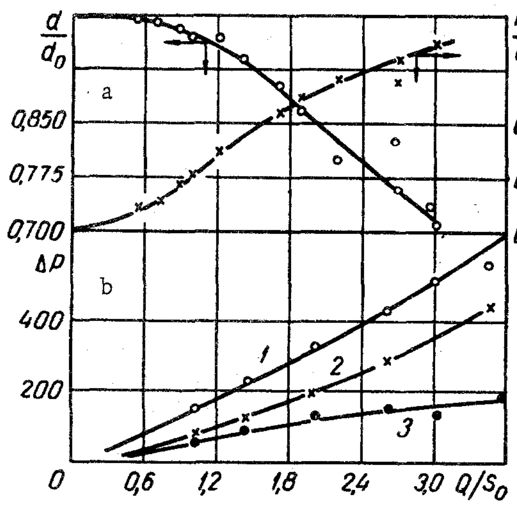

Fig. 4. Variation of a) h_c/d_0, d/d_0 and b) ΔP as a function of the gas flow velocity in the orifice: 1) ΔP_1; 2) ΔP_2; 3) ΔP_3.

The discharge geometry is governed mainly by the internal friction angle and depends also on the height of the column and the diameters of the vessel and orifice. The internal friction angle is determined by the packing density and the shape and other physical characteristics of the particles. As the absolute value of the internal friction increases, a gradular transition occurs from one form of motion to another. At given values of vessel and orifice diameters, the moment of transition is determined by the relation between the dimensions of the dynamic ellipsoid and the height of the column. Comparison of the height of a column of material that has not been forcibly packed into the vessel with the height of the dynamic ellipsoid (Fig. 1b) enables one to determine beforehand the form of discharge and the nature of particle motion in the vessel.

If the height of the dynamic ellipsoid and the column of loose material are the same, the ellipsoid emerges at the surface, and the discharge will be of the first or "normal" type. When $H_c \gg H_p$ the second "hydraulic" type of discharge is observed.

NOTATION

H_M, H_c, h_c, H_p, H_{pg} – heights of model, column of granular material, level at which actual discharge orifice is located, and of dynamic elipsoid for free discharge and gas counterflow; V_p – volume of dynamic ellipsoid; ε – eccentricity of dynamic ellipsoid; ξ – dynamic porosity; δ—thickness of layer surrounding dynamic ellipsoid; d_m, d_k, d_0, d – diameters of model, of a lump of material, of orifice, and of effective orifice for gas counterflow; r – radius of discharge funnel; s_0 – area of orifice of diameter d_0; Q – mass flow rate of air through orifice; ΔP_1, ΔP_2, ΔP_3 – pressure drops: over-all for discharge through the orifice and steady motion of column, in the stationary loosened column, and counterpressure exerted by gas on flow of particles through orifice; W_g, W_c – gas flow velocity in plane of orifice and rate of descent of column of material in vessel; W_0 – mass flow rate of granular material through orifice; α, α_1 – flow angles of loose material for free fall and in gas counterflow; β – angle of natural repose; γ – bulk weight of material; η, f – mobility and internal friction coefficient; g – acceleration due to gravity; Ψ – orifice form factor; K, m, n – coefficients taking account of particle size.

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